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EFFECT OF AN ELECTRIC CURRENT ON NECKING IN A TENSILE ROD

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Necking conditions in a tensile thermoviscoplastic rod with passage through it of an alternating electric current are studied. Modeling is performed with allowance for the complex constitutive relations for the rod material, heat transfer in the rod, and the current distribution across the section of the rod as a function of the current frequency (skin effect). The stability of uniform tension is examined by linear analysis of perturbations using the Routh–Hurwitz theory. The results were refined by nonlinear analysis taking into account the effect of the amplitude curve of perturbations on the stability of plastic deformation.

Introduction. In a previous paper [1], we calculated necking conditions in a tensile rod using a model (see the bibliography in [1]) that assumes instantaneous occurrence and stabilization of local thinnings in the sample at an early stage of strain localization. Stable localization of shape changes is associated with increase in the number of spontaneously formed necks at rather large strains. We believe that further progress in studies of necking should be aimed at development of methods for controlling deformation conditions in samples. One of such methods is the electroplastic treatment of samples aimed at reducing energy expenditures and preventing necking, in particular, in wire drawing [2, 3].

Spitsyn and Troitskii [3] assume that the effect of an electric current on the mechanical characteristics of a loaded solid is associated with Joule heating, the ponderomotive forces produced by the magnetic field of the current, and the "electron wind" due to electron scattering by dislocations (electron–plastic effect). These mechanisms facilitate plastic deformation at sites of concentration of mechanical stresses, and, hence, electrical treatment of samples can be considered a promising technology. Maksimov and Svirina [4, 5] calculated the effect of Joule heat on crack-propagation conditions. As regards the effect of an electric current on necking conditions in a tensile rod, we are aware only of a paper by Ruzanov et al. [6], in which it is concluded from calculations that a pulsed electric current does not influence necking but only exerts a general plasticizing effect on deformable samples. In our opinion, however, Ruzanov et al. [6] do not advance strong arguments in favor of the conclusions drawn, and the problem requires additional examination.

In the present paper, we restrict ourselves to necking conditions in a solid rod made of a thermoviscoplastic material with various strength and frequencies of the alternating electric current flowing through the rod.

1. Formulation of the Problem. The necking problem for a solid rod of density ρ_0 in uniaxial tension is formulated in [7]. We supplement the assumptions of [7] on deformation conditions in the rod by the assumption that the alternating electric current acts on the rod at constant potential difference U at the rod ends. The heating of the sample is taken into account by Joule heat and the Thomson effect [8]. Assuming that the initial cross section of the rod A_0 is homogeneous along the length, we obtain the following system of equations describing the behavior of the sample at large plastic strains:

$$\frac{\partial\varepsilon}{\partial t} = \mathrm{e}^{-\varepsilon}\frac{\partial v}{\partial X}, \qquad \rho_0\frac{\partial v}{\partial t} = \frac{\partial}{\partial X}(\sigma \mathrm{e}^{-\varepsilon}), \qquad C\frac{\partial\theta}{\partial t} = k\frac{\partial^2\theta}{\partial x^2} + \lambda j\frac{\partial\theta}{\partial x} + \gamma j^2 + \beta\sigma\frac{\partial\varepsilon}{\partial t}. \tag{1}$$

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Here v is the rate of displacement, ε is the strain, θ is the temperature, C is the heat capacity, k is the thermal conductivity, β is the fraction of plastic work converted to heat, λ is the Thomson coefficient [8], $\gamma = \gamma_0(1+\alpha(\theta-273))$, γ_0 is the initial specific resistance, α is the temperature resistance coefficient, and j is the current density at constant voltage $U = U_0$ on the sample:

$$j = \bar{j}_0 e^{\varepsilon}\, l_0 k_\omega(0,\theta^*)) \Big/ \int\limits_0^{l_0} k_\omega(\varepsilon,\theta)\, dX.$$

Here $k_\omega(\varepsilon,\theta) = (\gamma/\gamma_0)\, e^{2\varepsilon}(1 + (A_0 e^{-\varepsilon}\omega/(10^7\gamma))^2/12)$ in the case of a weak skin effect [8] and

$$k_\omega(\varepsilon,\theta) = \frac{\gamma}{\gamma_0} e^{2\varepsilon}(0.277 + 0.997(A_0 e^{-\varepsilon}\omega/(2\cdot 10^7\gamma))^{1/2})$$

in the case of a strong skin effect [8], $\bar{j}_0$ and U_0 are the current density and voltage on the sample at the initial time, and l_0 is the initial length of the sample.

The relation between the Eulerian coordinate x and the Lagrange coordinate X is given by

$$x = X + \int\limits_0^t v(X,\tau)\, d\tau. \tag{1a}$$

The function $\sigma = F_t^{-1}\psi(\theta,\varepsilon,\dot\varepsilon)$ specifies the nonlinear constitutive relation for the rod material. The Bridgman factor $F_t^{-1} = (1 + 2R_c/R)\log(1 + R/(2R_c))$ allows for the triaxial stress in the neck, and the local radius of the rod cross section R and the radius of the neck R_c are related by the formula $R_c = (1 + (\partial R/\partial x)^2)^{3/2}/(\partial^2 R/\partial x^2)$ [1].

We write the constitutive relation in the same form as in [7]:

$$\sigma = \mu F_t^{-1}\varepsilon^n \dot\varepsilon^m \theta^\nu. \tag{2}$$

Here μ, n, m, and ν are constants. In this case, for (1) the following initial and boundary conditions are assumed:

$$\begin{gathered} t=0: \quad \varepsilon = 0, \quad \dot\varepsilon = \frac{V}{l_0} = \dot\varepsilon^*, \quad v = \frac{V}{l_0}X = \dot\varepsilon^* X, \quad \theta = \theta^*, \\ X = 0: \quad v = 0, \quad \frac{\partial\theta}{\partial X} = 0, \qquad X = l_0: \quad v = V, \quad \frac{\partial\theta}{\partial X} = 0 \end{gathered} \tag{3}$$

(V, θ^*, and $\dot\varepsilon^*$ are constants).

2. Linear Analysis. We consider, at the time t_0, the homogeneous time-dependent solution ε_0, σ_0, v_0, θ_0, and F_{t0} of Eqs. (1) and (2) with initial and boundary conditions (3). As in [7], a small perturbation of this solution (nonhomogeneous solution) is written as

$$\begin{gathered} \varepsilon(X,t) = \varepsilon_0(t) + \delta\varepsilon(X,t) = \varepsilon_0(t) + \delta\varepsilon_0 e^{\eta(t-t_0)} e^{i\xi X}, \\ \sigma(X,t) = \sigma_0(t) + \delta\sigma(X,t) = \sigma_0(t) + \delta\sigma_0 e^{\eta(t-t_0)} e^{i\xi X}, \\ v(X,t) = v_0(X,t) + \delta v(X,t) = v_0(X,t) + \delta v_0 e^{\eta(t-t_0)} e^{i\xi X}, \\ \theta(X,t) = \theta_0(t) + \delta\theta(X,t) = \theta_0(t) + \delta\theta_0 e^{\eta(t-t_0)} e^{i\xi X}, \\ F_t(X,t) = F_{t0}(t) + \delta F_t(X,t) = F_{t0}(t) + \delta F_{t0} e^{\eta(t-t_0)} e^{i\xi X}, \end{gathered} \tag{4}$$

where $\delta\varepsilon$, $\delta\sigma$, δv, $\delta\theta$, and δF_t are the amplitudes of perturbations, $\eta = \delta\dot\varepsilon/(\delta\varepsilon)$ is a measure of perturbation growth, and ξ is a wavenumber. The choice of the nonhomogeneous solution (4) is based on the assumption that the amplitude of the perturbation is small compared to ε_0, σ_0, v_0, θ_0, and F_{t0}. Then, the Fourier series of the nonhomogeneous solution can be restricted to the first term of the series. This method is commonly used for stationary perturbed solutions, but it can also be employed for stability analysis of time-dependent solutions [7]. It is assumed in this case that the growth rate of perturbation is much higher than the growth rate of the homogeneous solution [9].

Substituting (4) into (1) and (2) and taking into account that $F_{t0} = 1$ and $\delta F_{t0} = -(A_0/(2\pi))\xi^2 e^{-2\varepsilon_0}\delta\sigma_0$, we obtain the following system of linear equations with the unknowns $\delta\varepsilon_0$, $\delta\sigma_0$, δv_0, and $\delta\theta_0$:

$$\Big(\frac{\partial\psi}{\partial\varepsilon} + \eta\,\frac{\partial\psi}{\partial\dot\varepsilon} + \frac{A_0}{2\pi}\,\xi^2 e^{-2\varepsilon_0}\sigma_0\Big)\delta\varepsilon_0 - \delta\sigma_0 + \frac{\partial\psi}{\partial\theta}\,\delta\theta_0 = 0,$$

$$-\sigma_0 e^{-\varepsilon_0} i\xi\delta\varepsilon_0 + e^{-\varepsilon_0} i\xi\delta\sigma_0 - \eta\rho_0\delta v_0 = 0, \qquad (\eta + \dot\varepsilon_0)\delta\varepsilon_0 - i\xi e^{-\varepsilon_0}\delta v_0 = 0,$$

$$(\eta\beta\sigma_0 + 2\gamma_0\bar{j}_0^2 e^{2\varepsilon_0}s^2(1+\alpha(\theta_0 - 273)))\,\delta\varepsilon_0$$

$$+\beta\dot\varepsilon_0\delta\sigma_0 + (\lambda\bar{j}_0 e^{\varepsilon_0} i\xi s + \alpha\gamma_0\bar{j}_0^2 e^{2\varepsilon_0}s^2 - C\eta - k\xi^2 e^{-2\varepsilon_0})\,\delta\theta_0 = 0.$$

Here $s = k_\omega(0,\theta^*)/k_\omega(\varepsilon_0,\theta_0)$. The roots of the characteristic equation of the present system determine the stability of the solution of the problem for uniform tension of the rod (homogeneous solution). The characteristic equation for the present system has the form

$$\eta^3 + (a_1' + ia_1'')\eta^2 + (a_2' + ia_2'')\eta + a_3' + ia_3'' = 0,$$

where

$$a_1' = \frac{1}{\rho_0 C}\Big[\xi^2\Big(e^{-2\varepsilon_0}C\,\frac{\partial\psi}{\partial\dot\varepsilon} + k\rho\Big) + \dot\varepsilon_0\rho\Big(C - \frac{\partial\psi}{\partial\theta}\,\beta\Big) - \rho_0\alpha\gamma_0\bar{j}_0^2 e^{2\varepsilon_0}s^2\Big], \quad a_1'' = -\frac{1}{C}\,\lambda\bar{j}_0\xi e^{\varepsilon_0}s,$$

$$a_2' = \frac{1}{\rho_0 C}\Big\{\xi^4 e^{-2\varepsilon_0}\Big(e^{-2\varepsilon_0}\sigma_0 C\,\frac{A_0}{2\pi} + k\,\frac{\partial\psi}{\partial\dot\varepsilon}\Big) + \xi^2\Big[e^{-2\varepsilon_0}\Big(\sigma_0\beta\,\frac{\partial\psi}{\partial\theta} + C\Big(\frac{\partial\psi}{\partial\varepsilon} - \sigma_0\Big)\Big) + \dot\varepsilon_0 k\rho\Big]$$

$$-\beta\rho\dot\varepsilon_0^2\,\frac{\partial\psi}{\partial\theta} - \Big(\frac{\partial\psi}{\partial\dot\varepsilon}\,\xi^2 e^{-2\varepsilon} + \rho_0\dot\varepsilon_0\Big)\alpha\gamma_0\bar{j}_0^2 e^{2\varepsilon_0}s^2\Big\},$$

$$a_2'' = -\frac{1}{\rho_0 C}\Big(\frac{\partial\psi}{\partial\dot\varepsilon}\,\xi^2 e^{-2\varepsilon} + \rho_0\dot\varepsilon_0\Big)\lambda\bar{j}_0\xi e^{\varepsilon_0}s,$$

$$a_3' = \frac{1}{\rho_0 C}\Big[\xi^6 e^{-4\varepsilon_0}\sigma_0\,\frac{A_0}{2\pi}\,k + \xi^4 e^{-2\varepsilon_0}k\Big(\frac{\partial\psi}{\partial\varepsilon} - \sigma_0\Big) + \xi^2 e^{-2\varepsilon_0}\sigma_0\,\frac{\partial\psi}{\partial\theta}\,\dot\varepsilon_0\beta$$

$$+2\gamma_0\bar{j}_0^2 s^2(1+\alpha(\theta_0 - 273))\,\frac{\partial\psi}{\partial\theta}\,\xi^2 e^{-2\varepsilon_0} - \Big(\frac{\partial\psi}{\partial\varepsilon} + \frac{A_0}{2\pi}\,\xi^2 e^{-2\varepsilon_0}\sigma_0 - \sigma_0\Big)\alpha\gamma_0\bar{j}_0^2\xi^2 s^2\Big],$$

$$a_3'' = -\frac{1}{\rho_0 C}\Big(\frac{\partial\psi}{\partial\varepsilon} + \frac{A_0}{2\pi}\,\xi^2 e^{-2\varepsilon_0}\sigma_0 - \sigma_0\Big)\lambda\bar{j}_0 s\xi^3 e^{-2\varepsilon_0}.$$

According to the Routh–Hurwitz stability theory, the solution of the problem is stable if all roots of the characteristic equation of the present linearized system have a negative real part. For this, it is necessary and sufficient that the matrix

$$\begin{pmatrix} 1 & -a_1'' & -a_2' & a_3'' & 0 \\ 0 & 1 & -a_1'' & -a_2' & a_3'' \\ 0 & 0 & a_1' & -a_2'' & -a_3' \\ 0 & a_1' & -a_2'' & -a_3' & 0 \\ a_1' & -a_2'' & -a_3' & 0 & 0 \end{pmatrix}$$

have positive inners [10].

To calculate the homogeneous strain rate $\dot\varepsilon_0$, we use the relation $\dot\varepsilon_0 = \dot\varepsilon^* e^{-\varepsilon_0}$, and the temperature of the rod at the stage of uniform deformation θ_0 is determined from the solution of the equation

$$C\,\frac{\partial\theta_0}{\partial t} = \gamma_0(1+\alpha(\theta_0 - 273))\bar{j}_0^2 e^{2\varepsilon_0}s^2 + \beta\sigma_0\,\frac{\partial\varepsilon_0}{\partial t}.$$

The stability boundary for the homogeneous solution of problem (1)–(3) corresponds to violation of the condition of positive inners of the characteristic matrix. Results of calculation of this boundary for the constants $\mu = 2.486\cdot 10^4$ MPa, $n = 0.52$, $C = 3.6\cdot 10^6$ J/(m$^2\cdot$K), $k = 15$ W/(m$\cdot$K), $\theta_0^* = 294$ K, $m = 0.002$,

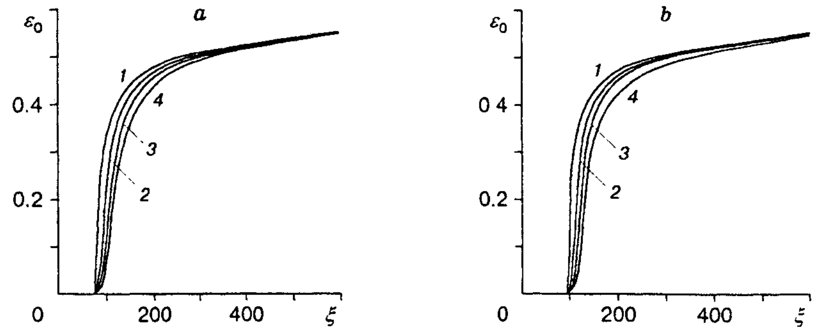

Fig. 1

$\nu = -0.5$, $\rho_0 = 7800$ kg/m^3, $A_0 = 4 \cdot 10^{-6}$ m^2 [7, 11], $\gamma_0 = 8.6 \cdot 10^{-8}$ $\Omega \cdot$m, $\lambda = -22.8 \cdot 10^{-6}$ V/K, $\alpha = 3.3 \cdot 10^{-3}$ K^{-1} [12], and $l_0 = 0.05$ m are shown in Fig. 1 for different values of the initial density $\bar{j}_0$ and frequency ω of the electric current [curves 1, 2, 3, and 4 correspond to $\omega = 0$, 500 kHz, 700 kHz, and 5 MHz, respectively; curves 1–3 refer to a weak skin effect and curve 4 refers to a strong skin effect; $\bar{j}_0 = 2 \cdot 10^7$ A/m^2 (Fig. 1a) and $\bar{j}_0 = 3 \cdot 10^7$ A/m^2 (Fig. 1b)]. The homogeneous solution is stable against the perturbation for ξ and ε_0 lying below the corresponding curves in Fig. 1a and b.

3. Nonlinear Analysis. Below, we perform a nonlinear stability analysis of necking in the tensile sample to refine the results of the linear analysis in Sec. 2. For this, to the homogeneous solution of the problem for homogeneous strain $\varepsilon_0 = \bar{\varepsilon}_0$ we add a perturbation of the strain of the form

$$\varepsilon_p = \varepsilon_0 \delta_0 \sin^2 (\xi(x - a)),$$

where δ_0 is the amplitude of the initial perturbation, $\xi = \pi/(b - a)$ is a wavenumber, and $a < x < b$, where a and b are the coordinates of the left and right boundaries of the perturbed region. Apparently, the coefficients of the Fourier series for the present perturbation decrease as N^{-3}, where N is the Fourier-coefficient number. The chosen form of perturbation ε_p ensures a small error in discarding Fourier components with $N > 1$ and is due to the necessity of comparing results of linear and nonlinear analyses. The latter allows us to examine necking conditions with variation in the perturbation amplitude δ_0. Results of calculations of the evolution of plastic strain (1)–(3) with perturbed initial conditions are given in Fig. 2 [$\dot{\varepsilon}^* = 1.66 \cdot 10^{-2}$ sec^{-1} and $\bar{\varepsilon}_0 = 0.45$], which shows the relative amplitude of the perturbation $\delta_\varepsilon(t) = (\max\limits_x \varepsilon(x, t) - \min\limits_x \varepsilon(x, t))/(\bar{\varepsilon}_0 \delta_0)$ $(0 < x < l_0)$ versus the homogeneous strain ε_0 for various values of the initial density $\bar{j}_0$ and frequency ω of the electric current and the wavenumber ξ (the notation is the same as in Fig. 1; $\bar{j}_0 = 2 \cdot 10^7$ (Fig. 2a and b) and $3 \cdot 10^7$ A/m^2 (Fig. 2c and d), and $\xi = 157$ (Fig. 2a and c), 314 m^{-1} (Fig. 2b and d)].

4. Discussion of Results. We analyze the results obtained. As shown in Fig. 1, at rather high strain rate $\dot{\varepsilon}^*$, the critical strain ε_{0c} depends greatly on the wavenumber ξ, particularly for its small values. According to the results of the linear analysis (Fig. 1) at constant voltage U, an increase in the current frequency ω leads to a decrease in the stability of the rod against the perturbation. In this case, the curves in Fig. 1b are below the curves in Fig. 1a for the same values of ω. Thus, an increase in the initial current density $\bar{j}_0$ favors earlier necking, and the effect of the electric current is most pronounced for small ξ. The shift to the right of the curves of $\varepsilon_{0c}(\xi)$ in Fig. 1 with increase in $\bar{j}_0$ s worth noting. At the same time, calculations show that when the temperature resistance coefficient is $\alpha = 0$, these shifts are absent and all curves issue from the point $\varepsilon_0 = 0$, $\xi = 0$.

According to the results of the nonlinear analysis (see Fig. 2), the perturbation ε_p first damps and then, upon reaching $\varepsilon = \varepsilon_{0c}$, the value of δ_ε begins to increase. In this case, for larger values of ξ, the increase in δ_ε begins later and it proceeds more slowly (the curves in Fig. 2a and c are above the curves in Fig. 2b and d for the same values of $\bar{j}_0$ and ω). The curves for large values of ω are above the curves for smaller ω, and the curves in Fig. 2a and b are below the curves in Fig. 2c and d for the same values of ξ and ω. This confirms the results of the linear analysis.

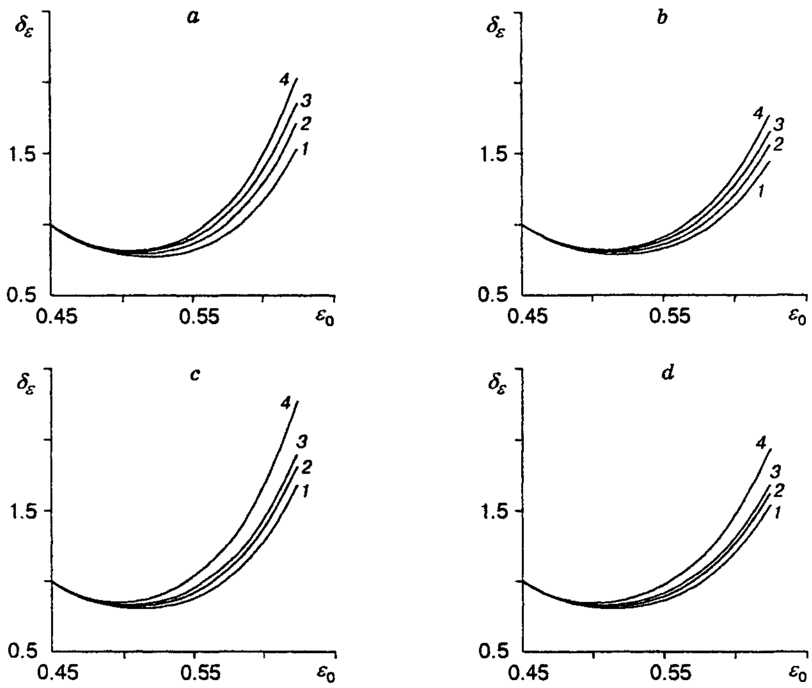

Fig. 2

The calculations performed for various amplitudes of the initial perturbation in the range $10^{-5} < \delta_0 < 10^{-2}$ showed that δ_ε practically does not depend on the value of δ_0. In addition, the Thomson effect is found to have little (less than 2% of ε_{0c}) effect on the stability of the deformable rod.

Calculations show that the action of the alternating electric current on the deformable rod facilitates necking in the rod. However, the current strength affects the critical necking strain to a greater extent than the current frequency. The amplitude of the strain perturbation and the Thomson effect influence necking only slightly.

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